THE INELASTIC PHOTON-ELECTRON COLLISIONS WITH POLARIZED BEAMS

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Abstract. We discussed the photoproduction of pair of charged particles $a\bar{a}\quad(a=e,\mu,\pi)$ as well as the double photon emission processes off an electron accounting for the polarization of colliding particles. In the kinematics when all the particles can be considered as a massless we obtain the compact analytical expressions for the differential cross sections of these processes. As the application of obtained results the special cases of production by circular and linear polarized photons are carry out.

We consider the lowest order inelastic QED processes in photon-electron interaction at high energies.We take into account the polarization of colliding photons and electron and consider the experimental setup,when the polarization of final particles is not measured.The interest to such kind processes is twofold. Linear e^+e^- high energy colliders(planned to be arranged [1, 2]) provide the possibility (using the backward laser Compton scattering) to obtain the high energy photon-electron colliding beams. The problem of calibration as well as the problem of important QED background are to be taken into account for this kind of colliders.The second important reason to investigate such reactions is the well known possibility to use the photoproduction of leptons pairs as a polarimeter process(see,for instance, [3] and references therein) .

Despite the fact that at high energy the bulk of particles are produced at very small angles, experimentally it is much easy to detect the particles produced at large angles.Thus we investigate the kinematic in which all invariants determining the matrix elements of the processes under consideration will be much bigger than masses of particles involved in the reaction. We consider the following set of inelastic reaction

$$\gamma(k,\lambda_\gamma)+e(p,\lambda_e)\to e(p',\lambda\prime_e)+a(q_-,\lambda_-)+\bar{a}(q_+,\lambda_+);\quad a=e,\mu,\pi$$

$$\gamma(k,\lambda_\gamma)+e(p,\lambda_e)\to e(p',\lambda_e\prime)+\gamma(k_1,\lambda_1)+\gamma(k_2,\lambda_2) \quad (1)$$

Here λ_i are the particle helicities
We will work in the kinematic when all the 4-vector scalar products defined by:

$$\begin{aligned} s &= 2pp', \quad s_1=2q_-q_+, \quad t=-2pq_-, \quad t_1=-2p'q_+, \quad u=-2pq_+, u_1=-2p'q_-, \\ \chi &= 2kp, \quad \chi'=2kp', \quad \chi_j=2k_jp, \quad \chi'_j=2k_jp' \quad j=1,2 \end{aligned} \quad (2)$$

are large compared with all masses:

$$\begin{aligned} s &\sim s_1\sim -t\sim -t_1\sim -u\sim -u_1\sim \chi_j\sim \chi'_j >> m^2; \\ p^2 &= p'^2=q_\pm^2=k^2=k_j^2-0. \end{aligned} \quad (3)$$

To obtain the cross sections of above reactions it is convenient to work with helicity amplitudes of corresponding processes [4]. The helicity amplitudes $M_{\lambda_\gamma\lambda}^{\lambda_-\lambda_+\lambda'}$ for lepton pair photoproduction, $M_{\lambda_\gamma\lambda}^{\lambda'}$ for a pair of charged pion production and $M_{\lambda_\gamma\lambda}^{\lambda_1\lambda_2\lambda'}$ for a two photon final state are defined as a usual matrix elements calculated with chiral states of photons and leptons.
The square of matrix element summed over the spin states of the final particles have the form of the conversion of the chiral matrix with the photon density matrix

$$\sum|M|^2=\frac{1}{2}Tr\begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22}\end{pmatrix}\begin{pmatrix} 1+\xi_2 & i\xi_1-\xi_3 \\ -i\xi_1-\xi_3 & 1-\xi_2\end{pmatrix}, \quad (4)$$

with the photon polarization vector $\vec{\xi}=(\xi_1,\xi_2,\xi_3)$ parameterized by Stokes parameters fulfilling the condition $\xi_1^2+\xi_2^2+\xi_3^2\le 1$.
The matrix elements of the chiral matrix m_{ij} are constructed from the chiral amplitudes of the process $M_{\lambda_\gamma\lambda_e}^{\lambda_-\lambda_+\lambda'}$ as

$$\begin{aligned} m_{11} &= \sum_{\lambda_-\lambda_+\lambda'}|M_{++}^{\lambda_-\lambda_+\lambda'}|^2, \quad m_{22}=\sum_{\lambda_-\lambda_+\lambda'}|M_{-+}^{\lambda_-\lambda_+\lambda'}|^2, \\ m_{12} &= \sum_{\lambda_-\lambda_+\lambda'}M_{++}^{\lambda_-\lambda_+\lambda'}(M_{-+}^{\lambda_-\lambda_+\lambda'})^*, \quad m_{21}=m_{12}^*. \end{aligned} \quad (5)$$

We put here only half of all chiral amplitudes which correspond to $\lambda_e=\frac{1}{2}$. The other half can be obtained from these ones by a space parity operation. The helicity amplitudes can be obtain using the relevant representations for photon polarization vector and Dirac spinors [5].As a result one obtains for the elements of chiral matrix for the processes under consideration the

following expressions:
1.Photoproduction of muon pair

$$
\begin{aligned}
m_{11} &= \frac{2w}{ss_1}(u^2+t^2), \quad m_{22} = \frac{2w}{ss_1}(u_1^2+t_1^2), &(6)\\
m_{12} &= -\frac{4(w_1 - iAw_2)}{(ss_1)^2}\Big[(ss_1)^2+(tt_1)^2+(uu_1)^2-2tt_1uu_1-ss_1(tt_1+uu_1)+4i(tt_1-uu_1)A\Big]
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \epsilon_{\mu\nu\rho\sigma}q_+^\mu q_-^\nu p^\rho p'^\sigma,\\
w &= -(\frac{q_+}{kq_+}-\frac{q_-}{kq_-}+\frac{p}{kp}-\frac{p'}{k.p'})^2,\\
w_1 &= \frac{w}{2}-\frac{4s_1}{\chi_+\chi_-}+\frac{\chi\chi\prime}{4s}\Big[\frac{w}{2}-\frac{2s}{\chi\chi\prime}-\frac{2s_1}{\chi_+\chi_-}\Big]^2,\\
w_2 &= \frac{2(\chi_++\chi_-)}{s\chi_+\chi_-}[\frac{w}{2}+\frac{2s}{\chi\chi\prime}-\frac{2s_1}{\chi_+\chi_-}] &(7)
\end{aligned}
$$

2.Electron pair photoproduction

$$
\begin{aligned}
m_{11} &= \frac{2w}{ss_1tt_1}\Big[t^3t_1+u^3u_1+s^3s_1\Big], \quad m_{22} = \frac{2w}{ss_1tt_1}\Big[t_1^3t+u_1^3u+s_1^3s\Big], &(8)\\
m_{12} &= \frac{4(w_1 - iAw_2)}{(ss_1tt_1)^2}\Big[(uu_1)^2-(ss_1)^2-(tt_1)^2\Big]\\
&\times \Big[\frac{1}{2}((uu_1)^2+(ss_1)^2+(tt_1)^2-2uu_1(ss_1+tt_1))+2iA(uu_1-ss_1-tt_1)\Big]
\end{aligned}
$$

3. Photoproduction of pions

$$
\begin{aligned}
m_{11} &= \frac{w}{2}tu, \quad m_{22} = \frac{w}{2}t_1u_1, &(9)\\
m_{12} &= \frac{w_1 - iAw_2}{ss_1}\Big[\frac{1}{2}(uu_1-tt_1)^2-ss_1(uu_1+tt_1)+2i(tt_1-uu_1)A\Big]
\end{aligned}
$$

The differential cross section of any pair production has the form

$$
\begin{aligned}
\frac{d\sigma}{d\Gamma} &= \frac{\alpha^3}{2\pi^2\chi}\Big[m_{11}+m_{22}+\xi_2\lambda_e(m_{11}-m_{22})-2\xi_3\mathrm{Re}(\mathrm{m}_{12})+2\xi_1\mathrm{Im}(\mathrm{m}_{12})\Big],\\
d\Gamma &= \frac{d^3p'}{\epsilon\prime}\frac{d^3q_-}{\epsilon_-}\frac{d^3q_+}{\epsilon_+}\delta^4(p+k-p'-q_+-q_-) &(10)
\end{aligned}
$$

where ξ_i and λ_e are Stocks parameters and target electron helicity.
The expressions (6)-(10) allows one to calculates the differential cross section of the processes of pair photoproduction off an electron with any polarization of colliding particles. Now it is a simple task to obtain from this

expressions the particular cases.
Let us consider the charged particles pair production with circular and linear polarization of photon from unpolarized target. The differential cross section for pair production by circularly polarized photons up to the factor ξ_2-the degree of circular (left or right) photon polarization coincide with the cross section in unpolarized case:

$$\begin{aligned}
\frac{d\sigma_{L(R)}}{d\Gamma} &= \frac{\alpha^3}{\pi^2\chi}\xi_{2L(2R)} w Z_{a\bar{a}}, \\
Z_{e\bar{e}} &= \frac{ss_1(s^2+s_1^2)+tt_1(t^2+t_1^2)+uu_1(u^2+u_1^2)}{ss_1tt_1}, \\
Z_{\mu\bar{\mu}} &= \frac{t^2+t_1^2+u^2+u_1^2}{ss_1}, \quad Z_{\pi\bar{\pi}} = \frac{tu+t_1u_1}{ss_1} \qquad (11)
\end{aligned}$$

Two of these quantities $Z_{e\bar{e}}, Z_{\mu\bar{\mu}}$ can be obtained from the relevant quantities obtained in papers [4] applying the crossing symmetry transformation. Much more interesting is the case of pair photoproduction by linear polarized photons for which the Stokes parameters are $\xi_2 = 0, \xi_1^2 + \xi_3^2 = 1$.In this specific case the square of full matrix element can be represented as the sum of diagonal term and non diagonal one, where the polarization parameters enter only the non diagonal term in the following form

$$Re\Big((\xi_3+i\xi_1)(w_1-iAw_2)(T_1+iAT_2)\Big) = \xi_3\Big(w_1T_1+w_2T_2A^2\Big)+\xi_1\Big(w_2T_1-w_1T_2\Big)A \qquad (12)$$

where the structures $T_{1(2)}$ depend on the type of created pair:
1. For the case of e^-e^+ production

$$\begin{aligned}
T_1 &= 4+8\frac{(uu_1-ss_1)(ss_1+tt_1)}{ss_1tt_1} - 4(\frac{tt_1-uu_1}{ss_1})^2, \\
T_2 &= 8[\frac{tt_1-uu_1}{s^2s_1^2}+\frac{1}{ss_1}-\frac{2}{tt_1}] \qquad (13)
\end{aligned}$$

2.In the case of $\mu_+\mu_-$ pair production

$$\begin{aligned}
T_1 &= 8\frac{tt_1+uu_1-ss_1}{ss_1} - 8(\frac{uu_1-tt_1}{ss_1})^2, \\
T_2 &= 16\frac{tt_1-uu_1}{(ss_1)^2} \qquad (14)
\end{aligned}$$

3. For the case of pion pair production:

$$\begin{aligned}
T_1 &= 4(\frac{uu_1-tt_1}{ss_1})^2 - 4\frac{uu_1+tt_1}{ss_1}, \\
T_2 &= \frac{uu_1-tt_1}{(ss_1)^2}. \qquad (15)
\end{aligned}$$

The same consideration can be done for double Compton effect.The differential cross section for this reaction in the general case, when the both primary particles are polarized can be cast in the following form:

$$\begin{aligned}\frac{d\sigma}{d\Gamma_\gamma} &= \frac{\alpha^3 s}{(2\pi)^2 D}\Big[\chi\chi'(\chi^2+\chi'^2)+\chi_1\chi_1'(\chi_1^2+\chi_1'^2)+\chi_2\chi_2'(\chi_2^2+\chi_2'^2) \\ &+ 4\xi_1 A(\chi_1\chi_2'-\chi_2\chi_1')-\xi_3(\chi_1\chi_2'+\chi_2\chi_1')(\chi\chi'-\chi_1\chi_1'-\chi_2\chi_2') \\ &+ \lambda_e\xi_2[\chi\chi'(\chi'^2-\chi^2)+\chi_1\chi_1'(\chi_1^2-\chi_1'^2)+\chi_2\chi_2'(\chi_2^2-\chi_2'^2)]\Big] \quad (16)\end{aligned}$$

with $D=\chi^2\chi'\chi_1\chi_1'\chi_2\chi_2'$ and $A=\epsilon_{\mu\nu\rho\sigma}k_2^\mu k_1^\nu p^\rho p'^\sigma$.
For unpolarized target the cross section of double Compton effect in the case of circularly polarized photon is:

$$\begin{aligned} d\sigma_{L(R)} &= \xi_{L(R)}\frac{2s\alpha^3}{\pi^2 D}Z_\gamma d\Gamma_\gamma, \\ Z_\gamma &= \chi\chi'(\chi^2+\chi'^2)+\chi_1\chi_1'(\chi_1^2+\chi_1'^2)+\chi_2\chi_2'(\chi_2^2+\chi_2'^2) \quad (17)\end{aligned}$$

with

$$d\Gamma_\gamma = \frac{d^3p_1'}{2\epsilon_1'}\frac{d^3k_1}{2\omega_1}\frac{d^3k_2}{2\omega_2}\delta^4(k+p_1-p_1'-k_1-k_2). \quad (18)$$

The summed on spin states square of the matrix element of the double Compton scattering process can as well be obtained from the ones for three photon annihilation of electron-positron pair,derived in [4].
For the case of double Compton effect under the linearly polarized photons one can obtain for non diagonal part of cross section which as in above case depend on Stoke's parameters the following expressions:

$$\begin{aligned} Re\ &\Big((\xi_3+i\xi_1)(T_1+iAT_2)\Big) = \xi_3 T_1-\xi_1 A T_2 \\ T_1 &= \frac{16s}{\chi\chi_1\chi_2\chi'\chi_1'\chi_2'}(\chi_1\chi_2'+\chi_2\chi_1')[\chi_1\chi_2'+\chi_2\chi_1'+s(s-\chi+\chi')], \\ T_2 &= \frac{8s}{\chi\chi_1\chi_2\chi'\chi_1'\chi_2'}(\chi_1\chi_2'-\chi_2\chi_1') \quad (19)\end{aligned}$$

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